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Optimization of raw material procurement plan in uncertain environment

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Abstract. In this paper, we study the optimization of raw material procurement plan in uncertain environment based on the Markov chain prediction model. Firstly, the Markov chain model is used to predict future demand and price based on the raw material demand and price data of a steel pipe enterprise. Then, we establish a raw material purchasing plan model with minimum cost of raw material procurement and inventory costs. Finally, we optimize the raw material procurement plan model by Genetic Algorithm.

Key words. Markov, purchase plan, Multi-cycle rolling purchase.

1. Introduction

Raw material costs, one-time procurement costs and inventory costs and many other aspects are taken into account in the early study on raw material procurement to determine the amount of raw materials procurement by making procurement overall costs a minimum in the case of the same raw material prices. While, raw material demand and price are susceptible in reality, so there are more frequent fluctuations and large changes in the uncertain situation. In this paper, we hold that the problem belongs to the multi-cycle optimization of raw material procurement in uncertain environment.

Currently, research on raw material procurement plan that take into account demand and price uncertainty has received the attention of some scholars. Early, Kingsman [1] considers the purchase price of goods to fluctuate over time in material procurement and proposes to use linear programming and dynamic programming to solve the uncertain problem. Arnold J, Minner S, and Eidam B [2] take into account that the purchase price, inventory costs and demand change over time in raw

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material procurement. But they assume that the purchase price obey a function of time. Xiongying Hu, Bin Hu and Jinlong Zhang [3] use the probability distribution function to describe the random price to solve the purchasing problem under seasonal random fluctuation in prices. In their study, a procurement model with the minimum cost of procurement is established to determine the optimal procurement plan. In the case of raw material procurement obeying the Markov process and demand subjecting to a Poisson distribution, Yang J and Xia Y [4] establish a procurement model with the goal of maximizing the total revenue to determine the optimal purchase quantity. Jingqiong Wu, Yun Pu and Jinqun Wu [5] establish a multi-period optimization model of raw material procurement for minimizing the unit purchase cost and determining the optimal purchase volume to ensure the supply of raw materials.

This paper optimize raw material procurement plan model by genetic algorithm to determine the optimal multiple cycles purchase plan.

2. Problem description

According to the actual situation of enterprises, the following assumptions can be made:

1) The purchase time of the enterprise is fixed;

2) The enterprise buys the current raw materials in the beginning of each period;

3) The purchase of raw materials are arrived in a day;

4) Different cycle of raw material demand and price are volatile, but remain unchanged in a period of time;

5) The probability of raw material demand or the first-order difference of raw material price from the current state to the future cycle of the state is the same.

The enterprise's raw materials are hot-rolled strip of different specifications. There are 350 species in total. In this paper, five raw material specifications of different width and thickness are selected as the experimental object.

3. Demand and price forecasts

(1) Raw Material Demand Forecast Based On Markov Chain

Markov predicted steps are as follows.

1) Data selection

This article is based on raw material demand data from February 2015 to July 2016.

2) State division

The raw material demand data is divided into five states in the method. Assuming that the minimum value of the demand data is $Mindata$ and the maximum value is $Maxdata$. State division method is different when problem is different. In this paper, the state partition model is:

State $E1$:$[Mindata, \frac{(Maxdata-Mindata)}{5}]$;

State $E2$:$[\frac{(Maxdata-Mindata)}{5}, \frac{(Maxdata-Mindata)}{5}*2]$;

State $E3$:$[\frac{(Maxdata-Mindata)}{5}*2, \frac{(Maxdata-Mindata)}{5}*3]$;
State $E4$:$[\frac{(Maxdata-Mindata)}{5}*3, \frac{(Maxdata-Mindata)}{5}*4]$;
State $E5$:$[\frac{(Maxdata-Mindata)}{5}*4, Maxdata]$.

Take specification 204.0X 2.75 as an example here.$Mindata = 993.6$ and $Maxdata$ $=4593.5$.According to the model above the five states of the specification are $[Mindata, 1713.6], [1713.6, 2433.6], [2433.6, 3153.5], [3153.5, 3873.5], [3873.5, Maxdata]$. According to the state division model, we can get the states of the demand of raw material specifications for each month from February 2015 to July 2016.The result of specification 204.0X 2.75 is shown in Table 1. And $E1, E2, E3, E4, E5$ represent states 1 to 5 respectively.

Table 1. Demand state change

year	January	February	March	April	May	June
2015	-	$E1$	$E2$	$E5$	$E5$	$E4$
2016	$E3$	$E1$	$E5$	$E5$	$E3$	$E3$
year	July	August	September	October	November	December
2015	$E3$	$E4$	$E3$	$E4$	$E4$	$E3$
2016	E4	-	-	-	-	-

3) Transfer probability calculation

According to Table 1 the first and second step transfer probability matrices are calculated to forecast demand states of August and September.

The initial state matrix represents the probability that the initial demand is in each state. As can be seen from Table 1, demand of July 2016 is in the $E4$ state so that the initial state matrix to forecast demand states of August and September is as follows:

$x0 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \end{pmatrix}$.

4) State prediction Assume that the initial state matrix is represented by $x0$. The prediction state matrices $x1, x2, ..., xt$ of the predicted object in the future period may be expressed as:

$x1 = P1 * x0, x2 = P2 * x0, \ldots, xt = Pt * x0$.

In the model, the predicted state matrix represents the probability that the object is in each state. For example $xt = [0 \quad 0.2 \quad 0.3 \quad 0.5 \quad 0]$ represents the probability of the cycle t in states 1 to 5 is: 0, 0.2, 0.3, 0.5, 0.

Based on the initial state matrix and the transition probability matrix, the demand states of the two cycles can by predicted. The state with the highest probability is the predicted state.

5) Value prediction The median of the interval represented by the predicted state is the predicted value. The forecast results of specifications 204.0X 2.75 are shown in Table 2.

Using different methods to predict the needs of five kinds of raw materials specifications and the comparative analysis of different forecasting methods is shown in Table 3.

Table 2. Raw material demand forecast results

Specification	Month	Actual demand (tons)	Predicted state (tons)	Forecast (tons)	Prediction error(%)
204.0X 2.75	8	2653	[2433.6,3153.5]	2793.6	5.30
	9	3039	[2433.6,3153.5],[3153.5,3873.5]	3153.5	3.77

Table 3. Prediction method comparison

method of prediction	Average prediction error(%)
Markov	6.16
Exponential smoothing method	15.76
BP neural network	12.62

(2) Raw material price forecast based on Markov Chain

After several tests, this article select from November 2015 to June 2016 price data to predict the July , August , September raw material prices because there are relatively large fluctuations in raw material prices and prices are affected by recent prices greater. First, we calculate the first-order difference of price according to raw material price characteristics. Then we apply the Markov Chain to predict.

The initial state matrix is:

$xt = (0 \quad 0 \quad 0 \quad 0.5 \quad 0)$.

The price forecast for the raw material specification 204.0X 2.75 is shown in Table 4 according to the state transition matrix and the initial state matrix.

Table 4. Raw material demand forecast results

specification	July			August			September		
	Actual value	Predictive value	Prediction error(%)	Actual value	Predictive value	Prediction error(%)	Actual value	Predictive value	Prediction error(%)
204.0X 2.75	2143.4	2075.4	3.17	2230.8	2174.4	2.53	2459.5	2395.1	2.62

Using different methods to predict the price of five kinds of raw materials specifications and the comparison of different prediction methods is in Table 5.

Table 5. Prediction method comparison

method of prediction	Average prediction error(%)
Markov	3.16
Exponential smoothing method	7.76
BP neural network	6.62

4. Purchasing plan optimization model and solution

(1) Procurement plan optimization model

The total cost of raw material procurement includes three aspects: raw material purchase costs, inventory costs and procurement transportation costs. In this paper, the total cost of raw material procurement includes raw material purchase costs and inventory costs because the cost of raw materials of the enterprise includes procurement and transportation costs.

1) Raw material purchase cost As can be seen from the above, the price of the raw material i is predicted to be $p_{0,i}$,$p_{1,i}$, . . . ,$p_{n-1,i}$ at time t_0,t_1, . . . ,t_{n-1}. Assume that the purchase amount of raw material i is $x_{0,i}$,$x_{1,i}$, . . . ,$x_{n-1,i}$. Let C_1 represents the purchase cost of raw materials in the purchasing plan and C_1 can be expressed as:

$$C_1 = \sum_{i=1}^{h}\sum_{j=1}^{n} x_{j-1,i}, p_{j-1,i}\,, \tag{1}$$

In the equation (1), n represents the total number of periods in the planned time domain and h represents the total number of raw materials.

What is needed to point out is, $x_{0,i}$,$x_{1,i}$, . . . ,$x_{n-1,i}$ are needed to meet the following constraints to meet the continuous production of enterprises and absence of stock requirements is not allowed:

$$\sum_{b=1}^{j} x_{b-1,i} + q_i \geq \sum_{b=1}^{j} d_b\,, \tag{2}$$

In the constraint (2), d_1,d_2, . . . ,d_n, indicate the needs of each cycle which is determined by the forecast above. And q_i represents the stock of raw material i at the beginning of the period.

Based on the inventory capabilities of the enterprise, $x_{0,i}$,$x_{1,i}$, . . . ,$x_{n-1,i}$ need to meet the following constraints:

$$\sum_{i=1}^{h} x_{j-1,i} \leq K_j\,. \tag{3}$$

In the constraints (3), j represents the cycle of j and K_j represents the inventory capacity of period j.

2) Raw material inventory costs

Let Q_j^A represents the raw material inventory at the beginning of the period j after purchasing and Q_j^B represents the stock of raw materials at the end of the period before purchasing. Q_j^A, Q_j^B can be expressed as:

$$Q_j^A = \sum_{i=1}^{h}(\sum_{b=1}^{j} x_{b-1,i} + q_i - \sum_{b=1}^{j-1} d_{b,h})\,, \tag{4}$$

$$Q_j^B = \sum_{i=1}^{h} (\sum_{b=1}^{j} x_{b-1,i} + q_i - \sum_{b=1}^{j} d_{b,h}) \,, \tag{5}$$

So that the average stock of raw materials in period j is:

$$\overline{Q}_j = \frac{Q_j^A + Q_j^B}{2}$$
$$= (\sum_{i=1}^{h} (\sum_{b=1}^{j} x_{b-1.i} + q_i - \sum_{b=1}^{j-1} d_{bh}) + \sum_{i=1}^{h} (\sum_{b=1}^{j} x_{b-1,i} + q_i - \sum_{b=1}^{j} d_{bh}))/2 \,, \tag{6}$$

If C_2 represents the raw material inventory cost in the raw material procurement plan, C_2 can be expressed as:

$$C_2 = \sum_{j=1}^{n} (\frac{Q_j^A + Q_j^B}{2}) * c_j \,. \tag{7}$$

In the equation (7), c_j represents the inventory cost of the raw material unit for the period.

3) Purchasing plan optimization model So Multi-cycle raw material procurement plan optimization model is as follows:

$$minC = C_1 + C_2 = (\sum_{i=1}^{h} \sum_{j=1}^{n} x_{j-1,i}, p_{j-1,i}$$
$$+ (\sum_{i=1}^{h} (\sum_{b=1}^{j} x_{b-1.i} + q_i - \sum_{b=1}^{j-1} d_{bh}) + \quad \sum_{i=1}^{h} (\sum_{b=1}^{j} x_{b-1,i} + q_i - \sum_{b=1}^{j} d_{bh}))/2) \,, \tag{8}$$

$$\sum_{b=1}^{j} x_{b-1,i} + q_i \geq \sum_{b=1}^{j} d_b, i = 1, 2, \ldots, h; j = 1, 2, \ldots, n \,, \tag{9}$$

$$\sum_{j=1}^{n} x_{j-1,i} + q_i = \sum_{j=1}^{n} d_i, i = 1, 2, \ldots, h \,, \tag{10}$$

$$\sum_{i=1}^{h} x_{j-1,i} \leq K_j, j = 1, 2, \ldots, n \,, \tag{11}$$

$$x_{ij} \geq 0 \,. \tag{12}$$

Where equation (8) represents raw materials procurement cost, and (9) shows production demand constraints, (10) represents that there is no surplus in the entire planning cycle, (11) shows that total weight of raw materials is less than inventory capacity, and finally (12) represents that raw material purchase weight is non-negative.

(2) Model solution

Since the model involves multiple variables and multiple cycles, it is difficult to use a conventional solution method, the Genetic Algorithm is chosen to solve the model.

1) Encoding

Using real matrix to encode and the chromosome matrix X is as follows:

$$X = \begin{pmatrix} x_{10} & x_{11} & \cdots & x_{1,n-1} \\ x_{20} & x_{21} & \cdots & x_{2,n-1} \\ \vdots & \vdots & \ddots & \vdots \\ x_{h0} & x_{h1} & \cdots & x_{h,n-1} \end{pmatrix}.$$

In the matrix X, x_{ij} represents the demand. The lines indicate different specifications and the columns show different cycles.

2) Determine the initial solution

In this paper, the initial solution is generated in order of cycles to make initial solution for each cycle which satisfies the demand for each cycle.

3) Evaluation of fitness values

Calculate the total purchasing cost of each chromosome as the fitness of the chromosome.

4) Choose

Calculate the probability according to the fitness value of each chromosome and choose the chromosome with the roulette method.

5) Cross

First, two cross bits are generated randomly. Then we calculate the sum of the numbers between the two intersections of the two chromosomes respectively. Finally, we calculate the new solution between the two cross bits of the two chromosomes according to the method of the initial solution. This ensures that the crossed chromosome is feasible.

6) Variation

The method of variation is similar to the method of cross.

Based on the traditional genetic algorithm we do some improvements in this paper.

In the Genetic Algorithms the population size is 100, the crossover probability is 0.9, the probability of mutation is 0.1 and the maximum number of iterations for genetic operations is 1500.

(3) The results and analysis The optimized procurement plan based on the results of the solution is shown in Table 6.

Table 6. Purchasing plan

specification	First cycle(tons)	Second cycle(tons)	The third cycle(tons)
204.0X 2.75	4396.0	4751.6	135.4
319.0X 2.0	3142.3	7176.1	39.6
520.0X 3.65	4150.2	971.7	0.1
520X3.90	6704.5	5590.3	149.2
260.0X 2.75	6294.7	1503.6	176.7

The comparison between the optimization results and the actual purchasing costs is shown in Table 7.

Table 7. Cost comparison analysis

specification	Actual purchase tons of cost (yuan / ton)	Actual Purchasing Total Cost (yuan)	Optimized purchase tons of cost(yuan / ton)	Optimized Purchasing Total Cost(yuan)	Total cost savings(yuan)	Percentage of cost reduction(%)
204.0X 2.75	3070.2	28501000	2920.7	27113000	2658000	4.87
319.0X 2.0	3114.7	32262000	2966.1	30722000	32262000	4.77
520.0X 3.65	3064.6	15697000	2820.6	14447000	15696999.9	7.96
520X3.90	3117.0	38788000	2903.1	36126000	38787999.9	6.86
260.0X 2.75	2997.7	23907000	2832.7	22591000	23906999.9	5.50

As can be seen from Table 7, the optimization method of purchasing plan proposed in this paper is significant for enterprises to reduce the cost of purchasing.

5. Conclusion

This paper first predicts the demand and price of raw materials based on Markov Chain. Then we build a raw material purchase plan optimization model based on the minimum cost of raw material purchase and inventory. Genetic Algorithm is used to solve the model which is complex. Finally, we generate a new raw material procurement plan. What's more, this paper compared with the existing procurement plan in the cost of tons, the amount of procurement and other aspects and the procurement costs are significantly reduced which shows that the optimization method of procurement plan proposed in this paper is feasible and effective.

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